Name————————————, **ID** ——————

Exam I, MTH 213, Spring 2012

Ayman Badawi

QUESTION 1. (i) **5 points** Let $A = \{1, 2, 3, 4\}$ and T be a partial order on A such that 3 is the maximum element of A under T and $1 \bigvee 2 = 4$. Then

T =

(ii) **5 points** Let $A = 1, 2, 3$ and T be a partial order on $P(A)$ such that whenever $x, y \in P(A)$, $(x, y) \in T$ iff $y \subseteq x$. Then

a) $\{1, 2\} \bigwedge \{3\}$ =

b) $\{2, 3\} \bigvee \{2\} =$

c) $\phi \bigwedge A =$

(iii) **3 points** Let $A = \{1, 2, 3\}$ and let T be a partial order on A. Given that T is also a function on A. Then $T =$

(iv) **6 points** Let T be a partial order on N such that whenever $a, b \in N$, $(a, b) \in T$ iff $a = bk$ for some $k \in N$. Then

a) $12 \bigwedge 8$ =

b) $6 \bigvee 9 =$

c) Minimum element of N under T

d) Maximum element of N under T

(v) **6 points** Let $A = \{0, 1, 1.5, 2, 3\}$. Define T on A such that $(a, b) \in T$ iff $a - b \in A$. Then

a) Find T.

b) Is T a partial order on A? EXPLAIN. If yes find the minimum element of A under T.

c) If your answer is yes in b) Find $1.5 \bigwedge 1 =$ and $1.5 \bigvee 2 =$

(vi) **5 points** Let $A = \{1, 2, 4, 6\}$. Construct an equivalence relation T on A such that A under T has exactly 3 distinct equivalent classes.

QUESTION 2. (i) **3 points** Find $(3466)_7 + (5201)_7$ =

(ii) **3 points** Convert $(2\ \underline{11}\ 1)_{12})$ to base 10

(iii) **4 points** Convert $(123)_{10}$ to base 5.

(iv) **4 points** Convert $(3\ \underline{23}\ \underline{15})_{27}$ to base 3

(v) **4 points** Convert $(244)_5$ to base 25

(vi) **6 points** Let $A = 25 \times 90 \times 36$ and $B = 24 \times 270 \times 12$.

a) Find $gcd(A, B)$

b) Find $LCM[A, B]$

QUESTION 3. a) **4 points** Prove that $\pi - 3.14$ is an irrational number.

b)**4 points** Prove that $\sqrt{60} + \sqrt{15}$ is an irrational number.

c)**4 points** Let A, B be sets. Prove that $A^c \cap B = B \setminus A$.

Faculty information

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Exam II, MTH 213, Spring 2012

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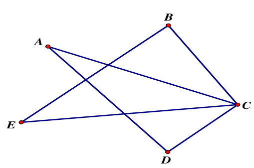
(Ten Questions, Each = 10, Total of points = 100)

QUESTION 1. Solve over Z
$2x \equiv 4 \pmod{6}, x \equiv 3 \pmod{4}, 3x \equiv 2 \pmod{5}$.

QUESTION 2. Find all solutions over Z_{25}: $10x = 15$

QUESTION 3. Find all integers that satisfy the following three properties : If each is divided by 8, then the remainder is 7. If each is divided by 4, then the remainder is 3. If each is divided by 36, then the remainder is 35.



QUESTION 4. If the graph is an Euler graph, then WRITE IT AS A CYCLE.

QUESTION 5. (quick answers)

(i) How many edges does $K_{4,7}$ have?

(ii) How many edges does K_{70} have?

(iii) Let $n = 21 \times 45 \times 56$. Then $\phi(n)$

(iv) As we explained in the class (counting Z) , what will be the integer in the 27 place?

(v) What is the place of -410?

QUESTION 6. USE MATH INDUCTION to prove that $7 \mid (2^{6n} - 1)$ for each $n \geq 1$.

QUESTION 7. We know that $gcd(45, 37) = 1$. Find TWO INTEGERS say F and W such that $1 = 45F + 37W$

QUESTION 8. Show that $|R| = |(-4, 4)|$ by constructing a bijection function from R ONTO $(-4, 4)$

QUESTION 9. Construct the zero-divisor graph of Z_{18}, i. e $G(Z_{18})$. Find $d(2, 12)$, $d(6, 14)$. Find the diameter and the girth of the graph.

QUESTION 10. Given $a_0 = -2, a_1 = -6, a_2 = 28$ and $a_n = -6a_{n-1} - 12a_{n-2} - 8a_{n-3}$ for each $n \geq 3$. Find a mathematical formula for a_n. Find the 7^{th} term of the sequence.

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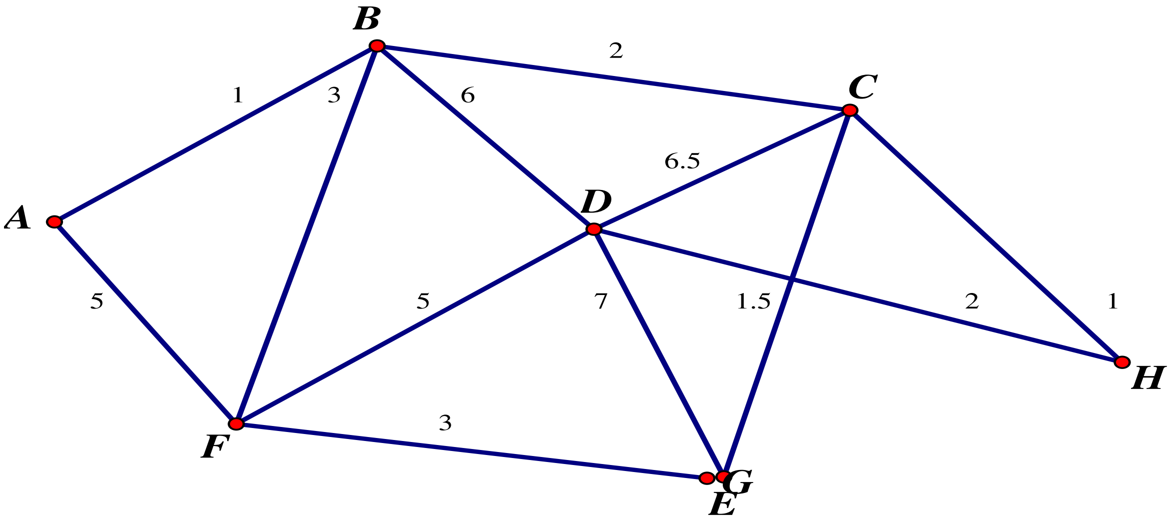
Final Exam , MTH 213, Spring 2012

Ayman Badawi

QUESTION 1. (15 points) a) Find the largest negative integer that satisfies the three conditions: If it is divided by 13, the remainder is 12. If it is divided by 9, the remainder is 8. If it is divided by 3, the remainder is 2.

b) Find the smallest positive integer that satisfies the three conditions as in (a)

QUESTION 2. (15 points) Consider the following weighted graph:



1) Is the graph Euler? Explain

2) Is the graph Khaldoon-Hamilton graph? Explain

3) Is the graph Hamilton? Explain

Find d(A, H)

USE THE ALGORITHM WE STUDIED TO FIND W-d(A, D)

QUESTION 3. (18 points) ((Write down T or F)

(i) If A is irrational, then A^3 is irrational

(ii) If A is rational, then $4\sqrt{A}+2$ is irrational.

(iii) If A is irrational, then $\sqrt{A}$ needs not be irrational

(iv) If for some integer A, we have $A \equiv 7 \pmod{36}$, then $A \equiv 3 \pmod{4}$

(v) $(-\infty, 12] \cap Q$ is a countable set

(vi) There exists an integer K such that $K \equiv 5 \pmod{11}$ and $K \equiv 0 \pmod{34}$

(vii) If A, B are sets, then $(A \setminus B) \cap (B \setminus A) = \phi$

(viii) If $A \subset B$, then $A^c \subset B^c$

(ix) $D = \{(a, a-6) \mid a \in N\}$ is a binary relation on N.

(x) If T is a binary relation on a set A that is not reflexive, then $a \in A$ implies $(a, a) \notin T$

(xi) If a binary relation is antisymmetric, then it is not symmetric

(xii) If T is a total order on a set A and $a, b \in A$ $(a \neq b)$, then $a \bigwedge b = a$ or $a \bigwedge b = b$

QUESTION 4. (8 points) Let $A = (-2, 6)$ and $B = [-4, 20] - \{8, 11\}$. Show that $|A| = |B|$ by constructing a bijection function
$F : A \to B$. Write down the equation (s) of F.

QUESTION 5. (8 points) Let $A = \{a, b, c, d, e, f\}$ and T be a partial binary relation on A such that
$T = \{(a, a), (b, b), (c, c), (d, d), (e, e), (f, f), (a, d), (d, e), (a, e), (b, e), (b, c), (c, f), (b, f)\}$
a) Find $b \vee d$

b) Find $c \wedge e$

c) Find $a \vee b$

c)Find the Minimum and the Maximum of A under T if they exist

QUESTION 6. (8 points) Use math INDUCTION to Prove that $2^n \leq n!$ (where $n!$ is read "n factorial") for every $n \geq 4$

QUESTION 7. (12 points) a) Find $(174)_8 + (635)_8$

b) Let $n = 33 \times 77 \times 24$. Find $\Phi(n)$.

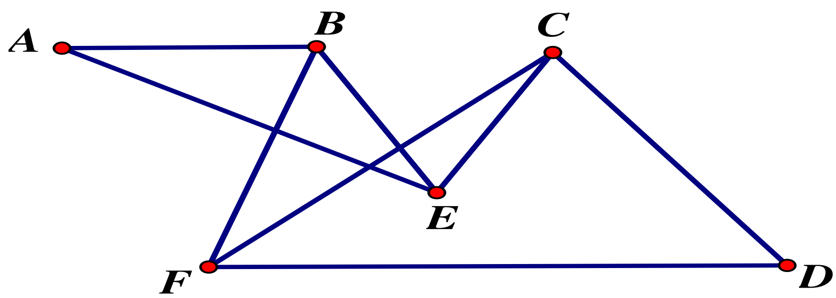
c) Convert 147 to base 5

d) Convert $(566)_7$ to base 49

e) Is there a graph with 5 vertices such that $deg(v_1) = deg(v_2) = deg(v_3) = 3$ and $deg(v_4) = deg(v_5) = 6$? If yes, is it an Euler Graph?

QUESTION 8. (8 points) Construct the total graph of $T(Z_9)$

QUESTION 9. (8 points) Consider the graph below



Is the graph a planar? if yes, then EXPLAIN by REDRAWING the graph

Is the graph Hamilton? If yes Explain

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